SPARC / MoFaCTS

TutorScript: the instructional cycle

Categorized dynamic instruction • source-audited review • 21 October 2026

THE LESSON ILLUSTRATED: ESTIMATION AND CONFIDENCE INTERVALS

The lesson defines seven knowledge components (KCs) and three misconceptions. These labels identify the targets used in the cycle and in the complete tutor moves below.

SEVEN KNOWLEDGE COMPONENTS

KC E1 Repeated-sampling interpretation assumes random samples and the same interval-construction method across repetitions.

KC E2 The confidence level describes the long-run proportion of constructed intervals that contain the true population mean.

KC E3 A computed confidence interval either contains the fixed true population mean or it does not.

KC A Construct a confidence interval by extending critical value × standard error on each side of the parameter estimate.

KC B With confidence level and population variability held fixed, margin of error decreases in proportion to $1/\sqrt{n}$.

KC C When population standard deviation is unknown and estimated from the sample, a mean interval uses a t multiplier with n − 1 degrees of freedom rather than the known-σ normal multiplier.

KC D The normal-approximation proportion interval requires sufficiently many observations in both outcome categories. A large total sample alone is insufficient.

KC-LED STARTUP

The authored starting KC is KC A. Select a move for that KC; each question supplies its own necessary context.

THREE MISCONCEPTIONS

Statements to detect, not correct claims.

M1 There is a 95% probability that the true mean lies inside this specific interval.

M2 The interval means 95% of individual observations fall between 42 and 50.

M3 The interval means 95% of sample means, future sample means, or future confidence intervals will fall between 42 and 50.

LESSON SETUP · Orient and save the maximum-relationship Hamiltonian path

Author chooses the starting KC → embed the KC texts → build the complete relationship graph → solve the maximum-weight Hamiltonian path with that KC fixed first.

Maximize Σ relationship(KC_i, KC_{i+1}), visiting every KC exactly once.

The author sets the orientation through the starting KC. The saved path orders KCs; it does not select instructional productions. No relationship cutoff or heuristic substitute.

1 Select one instructional target

At startup or a KC boundary, select the first KC below the coverage criterion in the saved path. Skip KCs already covered through incidental credit.

Continue an unfinished active target. An eligible misconception can interrupt a KC; repair has priority. At completion, save feedback and end the lesson.

Confidence-interval criteria: KC coverage ≥ 0.8; misconception trigger ≥ 0.2.

2 Choose the production for that target

Four compositions × four response formats = 16 alternatives

Assistance a(m), in logits

	Text	Cloze	Multiple choice	True/false
Question	0.5	0.75	1	1.25
Explanation → question	1.5	1.75	2	2.25
Explanation → written support → question	2.5	2.75	3	3.25
Explanation → visual support → question	2.5	2.75	3	3.25

STUDENT SELECTS

Choose Same KC or Next KC, a composition, and a response format. Composition × format gives 16 alternatives. These choices do not score an answer.

AGENT: BOLTZMANN SELECTION

$$u(m) = -|\theta + a(m) - \text{logit}(0.8)|$$

$$P(m) = \exp[u(m)/T] \,/\, \Sigma_j \exp[u(j)/T]$$

T = 0.2. Sample among moves for this target, favoring expected success near 0.8.

3 Realize and present the selected move

The KC's authored category guides learning processes: fact (memory/fluency), model (understanding/sense-making), rule (induction/refinement), or their combinations.

Retrieve suitable saved primitives or generate missing content. Written support supplies an example or comparison; visual support supplies an image or a video passage.

Display order: explanation → support → question, omitting unrequested components.

Ready components appear in order; an image must load before the question is shown.

STUDENT RESPONSE

4 Assess the answer and save the evidence

Evaluate the current KC and its three strongest graph neighbors, plus up to four related misconceptions (retaining an active misconception). Unassessed scores carry forward.

Judge the meaning elicited by the actual question and response format. Credit student evidence; supplied answers and repeated wording do not earn new credit.

q = KC coverage, or 1 − misconception support

θ = max[logit(0.2), logit(seed) + Σ saved Δlogit] seed = 0.2

5 Give feedback; continue or stop

Feedback addresses the assessed answer. Student condition: show feedback before offering the next menu. Agent condition: feedback and next-move generation may overlap.

In both conditions, save feedback before saving or displaying the next instruction. Resume reuses accepted assessment, decisions, and completed components.

If complete: final feedback → Continue. Otherwise: return to target selection.

THREE COMPLETE INSTRUCTIONAL CYCLES

Unanswered instruction → full question with the learner's answer highlighted → matching feedback

Explanation and question

Multiple choice

1 INSTRUCTION

For a normal-approximation confidence interval for a population proportion, check the observed counts in **both** outcome categories: successes and failures. A common rule requires at least 10 of each. A large total sample does not guarantee this condition: if one outcome is rare, its count may still be too small for the normal approximation.

A survey records 1,496 "yes" responses and 4 "no" responses. Researchers then collect 100 additional responses, all "yes." Using the guideline of at least 10 observations in each outcome category, does the expanded sample meet the outcome-count condition for a normal-approximation proportion interval?

- Yes, because the 100 additional responses exceed the minimum count of 10.
- No, because the two outcome counts must be equal.
- No, because the expanded sample still contains only 4 "no" responses.
- Yes, because the expanded sample contains more than 1,000 observations.

2 LEARNER'S ANSWER

A survey records 1,496 "yes" responses and 4 "no" responses. Researchers then collect 100 additional responses, all "yes." Using the guideline of at least 10 observations in each outcome category, does the expanded sample meet the outcome-count condition for a normal-approximation proportion interval?

- A Yes, because the 100 additional responses exceed the minimum count of 10.
- B No, because the two outcome counts must be equal.
- ✓ **No, because the expanded sample still contains only 4 "no" responses.**
- D Yes, because the expanded sample contains more than 1,000 observations.

3 TUTOR FEEDBACK

Correct—the additional "yes" responses leave the "no" count at 4, below the required 10. Both outcome categories must meet the guideline, so the expanded sample still fails the condition despite its large total size.

With a written comparison

Fill in the blank

1 INSTRUCTION

For a confidence interval for a population mean, estimating the unknown population standard deviation with the sample standard deviation adds uncertainty. Use a **t multiplier with n − 1 degrees of freedom** to account for it; reserve the normal z multiplier for known σ. The Comparison highlights this distinction while holding sample size fixed.

Comparison

Two samples each contain 20 observations and are used to construct a confidence interval for a population mean. Assume the conditions for each interval are satisfied.

Case	Source of standard deviation	Multiplier	Reason
Population standard deviation is known	Known population value σ	Normal multiplier z^*	The population standard deviation does not need to be estimated.
Population standard deviation is unknown	Sample estimate s	t^* with 19 degrees of freedom	Estimating σ from the sample requires a t multiplier, with degrees of freedom n − 1 = 20 − 1.

The decisive feature is whether σ is known or estimated from the sample—not sample size alone. When σ is unknown, the mean interval is $\bar{x} \pm t^* s/\sqrt{n}$, using n − 1 degrees of freedom.

A researcher constructs a confidence interval for a population mean from 36 observations, using the sample standard deviation because the population standard deviation is unknown. Assuming the interval conditions are satisfied, the appropriate multiplier is ____.

2 LEARNER'S ANSWER

A researcher constructs a confidence interval for a population mean from 36 observations, using the sample standard deviation because the population standard deviation is unknown. Assuming the interval conditions are satisfied, the appropriate multiplier is **a t multiplier with 35 degrees of freedom**.

3 TUTOR FEEDBACK

Correct. Because the unknown population standard deviation is estimated using the sample standard deviation, use a t multiplier with n − 1 = 36 − 1 = 35 degrees of freedom.

With visual support

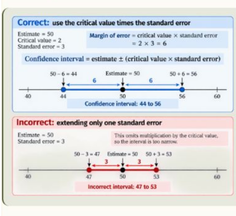
True / false

1 INSTRUCTION

Constructing a confidence interval gives a range around a parameter estimate that reflects uncertainty. First calculate the **margin of error = critical value × standard error**. Then subtract that margin from the estimate for the lower endpoint and add it for the upper endpoint:

Confidence interval = estimate ± (critical value × standard error).

Use the full margin of error on each side, rather than the standard error alone.



For a parameter estimate of 80, a standard error of 4, and a critical value of 2.5, the confidence interval is [70, 90].

- True
- False

2 LEARNER'S ANSWER

For a parameter estimate of 80, a standard error of 4, and a critical value of 2.5, the confidence interval is [70, 90].

- ✓ **True**
- ○ False

3 TUTOR FEEDBACK

Correct. The margin of error is 2.5 × 4 = 10. Extending it on each side of 80 gives 80 ± 10, or [70, 90].

Presentation note: learner panels repeat the full question and highlight the submitted correct response for clarity. These learner panels are figure-specific; instruction and feedback panels are captures of the actual interface.

Each column is one complete local synthetic test cycle, captured before and after answering on 21 October 2026. The original October 13 figure is preserved. Scope: categorized agent/student v2 policies with startingKC configured.

Logit(p) = ln[p/(1−p)]; probabilities are bounded to 0.0001–0.9999 for conversion.